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Glueball and gluelump spectrum in abelian projected QCD *

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We study glueball and gluelump spectra calculated after abelian projection in both quenched and $N_f = 2$ full QCD. The abelian projection is made after MA gauge fixing. We demonstrate that both spectra can be recovered despite the problem with positivity. We suggest the interpretation of some of the gluelump states in the language of the abelian projected theory.

1. INTRODUCTION

The effective infrared theory obtained after abelian projection must reproduce the low mass hadron spectrum at least in qualitative agreement with the real spectrum. To check this in the maximally abelian (MA) projection [1] we calculate glueball and gluelump spectrum in the abelian projected (AP) SU(3) theory. We also discuss results for QCD with dynamical fermions. The glueball spectrum in AP $SU(2)$ theory has been studied in [2] and good agreement with the $SU(2)$ spectrum has been found. The low hadron masses have been computed in AP $SU(3)$ [3]. This study, though being limited in precision, allows to draw conclusion about qualitative agreement with the spectrum of the unprojected theory. There is another motivation of our work. It has been claimed [4] that since abelian projection breaks $SU(3)$ invariance (even global) the 'new hadronic' states must appear in a theory which are absent in the experimental spectrum. We will suggest a solution of this problem.

2. SIMULATION DETAILS

We follow the standard procedure of abelian projection for $SU(3)$ in MA gauge [1]. To fix MA gauge a simulated annealing algorithm [5] has been employed with the aim to reduce effects of Gribov copies. We generated one gauge copy per configuration. Our computations have been done on a $16^3 \cdot 32$ lattice. In the quenched case we used $O(400)$ configurations at $\beta = 6.0$. For the $N_f = 2$ full QCD $O(300)$ configurations at $\beta = 5.29, \kappa = 0.1350$ generated by QCDSF [6] have been used. These two sets have roughly equal lattice spacing with $r_0/a \approx 5.3$.

3. GLUEBALLS IN QUENCHED/FULL AP QCD

Let us introduce lattice gauge field $U_{x,\mu} \in SU(3)$ and abelian projected field $u_{x,\mu} = \mathrm{diag}\{e^{i\theta^1_{x,\mu}}, e^{i\theta^2_{x,\mu}}, e^{i\theta^3_{x,\mu}}\} \in U(1) \times U(1)$. The glueball correlator has a general form

$$\Gamma(t) =< \mathrm{Tr}G(0)\mathrm{Tr}G^\dagger(t) > - < \mathrm{Tr}G >^2 \qquad (1)$$

The zero momentum operator

$$G(t) = \sum_{\vec{x}} G(\vec{x}, t) \qquad (2)$$

$$G(\vec{x}, t) = \sum_C \left(U(C_x) \pm U^\dagger(C_x)\right), \; U(C) = \prod_{l \in C} U_l$$

belongs to one of the three representations of the cubic group: $\mathrm{A}^{++}, \mathrm{E}^{++}, \mathrm{T1}^{+-}$. In (3) closed paths C and sign are chosen properly to get particular representation. To obtain the corresponding projected correlator we make the following substitution in (3): $U(C) \rightarrow u(C)$. We use

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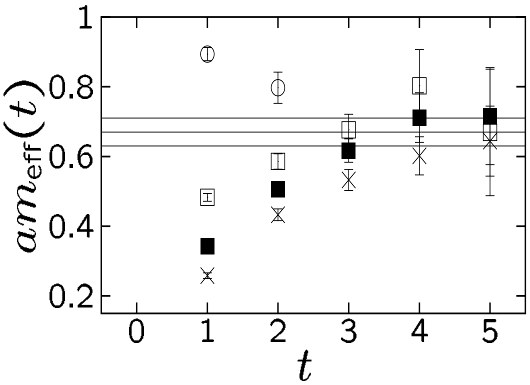

Figure 1. 0^{++} *glueball effective mass for the following operators:* 1×1 *unsmeared* (○), 2×2 *smeared* (□), 4×4 *smeared* (■), 3 *levels of fuzzing* ($\times$).

square loops of a size up to 8×8 and apply smearing or fuzzing to $u_{x,i}$. Finally, the effective mass is extracted as usual:

$$am_{\text{eff}}(t) = -\log\left[\frac{\Gamma(t+1)}{\Gamma(t)}\right] \qquad (3)$$

In Fig. 1 we present our results for projected $m_{\text{eff}}(t)$ for 0^{++} glueball. The straight lines show central value and error bars of the 0^{++} glueball mass obtained at this β value in [7] in the unprojected theory. One can see unusual behaviour of $m_{\text{eff}}(t)$ for smeared/fuzzed operators. This is due to lack of reflection positivity for gauge noninvariant operators used in our computations, as has been discussed in [2]. We found consistency between our results obtained with various operators which assures us that our computation of the glueball masses is meaningful. Good agreement with results of [7] can be seen from Fig. 1. Similar agreement was found for 2^{++} and 1^{+-} glueballs. We repeated our computation on full QCD configurations and obtained results consistent within error bars with results for the quenched case.

4. GLUELUMPS IN QUENCHED/FULL AP QCD

The gluelump was introduced in [8]. It is not a physical particle. It can be seen as a glueball with one gluon infinitely massive or as a static adjoint quark with color screened by dynamical gluon field. Its lowest energy determines a scale where adjoint string should break. The gluelump correlator has the following form:

$$\Gamma(t) = \mathrm{Tr}\left[G(\vec{x},0)\lambda_a\right]\ S_{ab}(\vec{x},t)\ \mathrm{Tr}\left[G(\vec{x},t)\lambda_b\right] \qquad (4)$$

$$S(\vec{x},t) = \prod_{\tau=1}^{t} U^{adj}_{(\vec{x},\tau),0}$$

The gluelump spectrum has been computed both in $SU(2)$ [9] and in $SU(3)$ [10]. It is worth noting that 1^{+-} and 1^{--} gluelump correlators coincide with the gauge invariant correlators of the magnetic and electric fields $< B_i^a(x)S_{ab}(x,y)B_i^b(y) >$, $< E_i^a(x)S_{ab}(x,y)E_i^b(y) >$ [11], where S_{ab} is a Schwinger line. In the stochastic vacuum model [12] the field strength correlator has exponential decay and determines the gluon correlation length T_g [12]. Within the scope of this model T_g is a fundamental parameter. The field strength correlator has been computed on the lattice. The lattice results for T_g can be summarized as follows: in $SU(2)$ $T_g = 0.15 \div 0.2$ fm [13], in $SU(3)$ $T_g = 0.1 \div 0.2$ fm [14]. The ground state energy of the gluelump, E_g, is related to T_g [11]:

$$\frac{1}{T_g} = m_g,\ m_g = E_g - \text{divergent selfenergy} \qquad (5)$$

The abelian projected gluelump correlator is:

$$\tilde{\Gamma}(t) = \sum_{\alpha=3,8} \mathrm{Tr}\left(\tilde{G}(\vec{x},0)\lambda_\alpha\right) \mathrm{Tr}\left(\tilde{G}(\vec{x},t)\lambda_\alpha\right)$$

Note the absence of the Schwinger line in the projected correlator. This implies that the energy computed from this correlator has no divergent part. Recently a connection between the field strength correlator and the dual photon propagator of an effective infrared QCD (dual Abelian Higgs model) has been suggested [15]. If this is true, then the dual photon mass is equal to m_g introduced above. In this sense we can say that the dual photon corresponds to the 1^{+-} gluelump and thus is not observable. We computed correlators $\tilde{\Gamma}(t)$ for $1^{+-}, 0^{++}, 2^{++}$ gluelumps using the same techniques as in the computation of the projected glueball spectrum. To compare with results of [10] we need to subtract the divergent part from

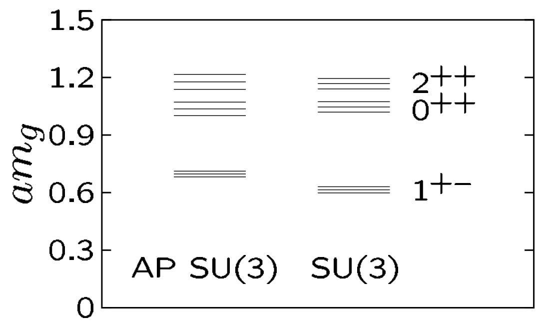

Figure 2. *Comparison of the gluelump spectra obtained with and without abelian projection.*

these results. This we make in the way applied to $SU(2)$ gluelumps in [9]: $m_g = E_g - \frac{1}{2}V_0^{adj}$, where V_0^{adj} is a selfenergy of the static adjoint quark-antiquark pair. To estimate V_0^{adj} we use the relation $V_0^{adj} = \frac{9}{4}V_0^{fund}$, which holds according to results of [16], and take the value $V_0^{fund} = 0.63(2)$ from [17]. Our results presented in Fig. 2 show good qualitative agreement with the $SU(3)$ spectrum of gluelumps obtained in [10]. Similar to the glueballs case the gluelump spectrum in AP QCD agrees well with results depicted in Fig. 2.
It is easy now to suggest the interpretation of the 'new hadron' states considered in [4], namely the states generated by operators $\bar{\psi}(x)\lambda_{3,8}\psi(x)$. They are abelian projections of adjoint-mesons [10] determined by the correlator

$$\Gamma(t) = \mathrm{Tr}\,[G_a(\vec{x},0)]\,S_{ab}(\vec{x},t)\mathrm{Tr}\,[G_b(\vec{x},t)] \qquad (6)$$

$$G_a(x) = \bar{\psi}(x)\lambda_a\psi(x)$$

After abelian projection the correlator is:

$$\tilde{\Gamma}(t) = \sum_{\alpha=3,8} \mathrm{Tr}\,[G_\alpha(\vec{x},0)]\,\mathrm{Tr}\,[G_\alpha(\vec{x},t)] \qquad (7)$$

The adjoint-baryons and their abelian projection can be constructed analogously. Note that we always use Weyl invariant operators since our gauge fixing does not break Weyl invariance.

5. CONCLUSIONS

Our results for glueball and gluelump spectra in AP $SU(3)$ show good qualitative agreement with corresponding results obtained in $SU(3)$. To make conclusion about how good agreement is quantitatively the extrapolation to the continuum limit would be necessary. Similar computations made in AP full QCD at $m_\pi/m_\rho = 0.76$ have not revealed any essential changes in the spectrum. We proposed solution of the problem of 'new hadrons', raised in [4]: these states are abelian counterparts of gluelumps (or adjoint mesons/baryons) and thus cannot be detected in experiments.
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